

7. Luminescent Materials

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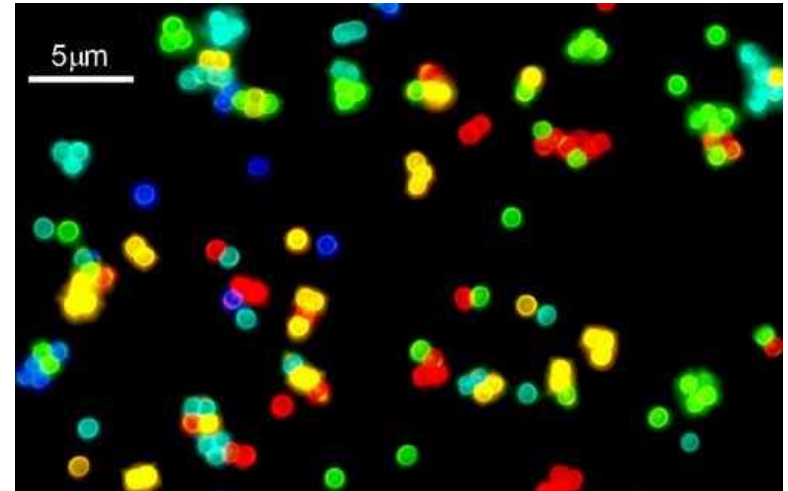
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Cd(S,Se) Quantum dots



7.1 History

Some milestones

- **Stone of Bologna: Barit (Galilei 1600)**
- **Discovery of phosphors (phosphorescence) (Brand 1669)**
- **First phosphor by reaction of shells with sulfur (Canton 1768)**
- **Application of a phosphor in combination with a Hg-discharge (Becquerel 1859)**
- **Patent on the use of CaWO_4 in fluorescent lamps (Edison 1896)**
- **Fluorescent lamps with $\text{MgWO}_4 + (\text{Zn,Be})_2\text{SiO}_4:\text{Mn}$ (GE 1938)**
- **Development of $\text{Ca}_5(\text{PO}_4)_3(\text{F,Cl}):\text{Sb,Mn}$ (McKeag 1942)**
- **$\text{ZnS}:\text{Ag}$, $(\text{Zn,Be})_2\text{SiO}_4:\text{Mn}$ and $\text{Zn}_3(\text{PO}_4)_2:\text{Mn}$ for first color CRT (1958)**
- **Fluorescent lamps with Eu- and Tb- phosphors (Verstegen 1974)**
- **First oxidic afterglow pigment $\text{SrAl}_2\text{O}_4:\text{Eu,Dy}$ (Nemoto 1993)**
- **Nitride phosphors (Schnick 1995)**
- **$\text{K}_2\text{SiF}_6:\text{Mn}^{4+}$ as red line emitter for LEDs (GE 2006)**
- **Transparent ceramics converter for LEDs (Philips 2007)**
- **Narrow band red nitride phosphor $\text{Sr}[\text{LiAl}_3\text{N}_4]:\text{Eu}$ (Schnick 2014)**



Credit: Harald Biecker



7.2 Definition and Working Principle

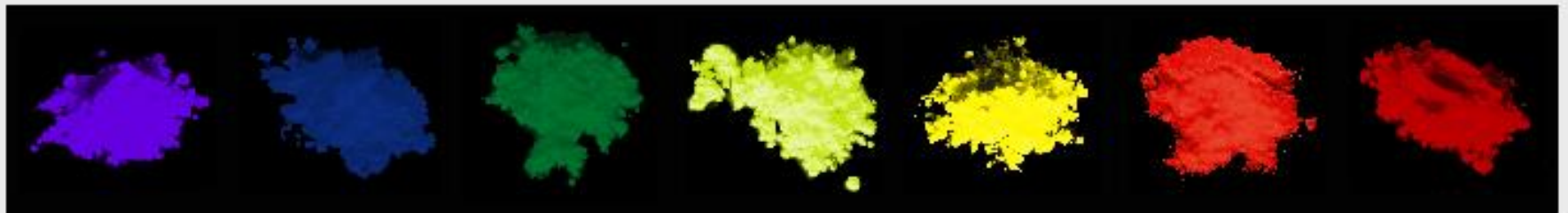
Definition

A phosphor is a micro-or nanoscale (in)organic pigment, that after excitation by radiation (NIR-, VIS-, UV-, X-ray-, gamma-), high-energy particles or matter vibrations (phonons), emits electromagnetic radiation beyond thermal equilibrium.

Under daylight



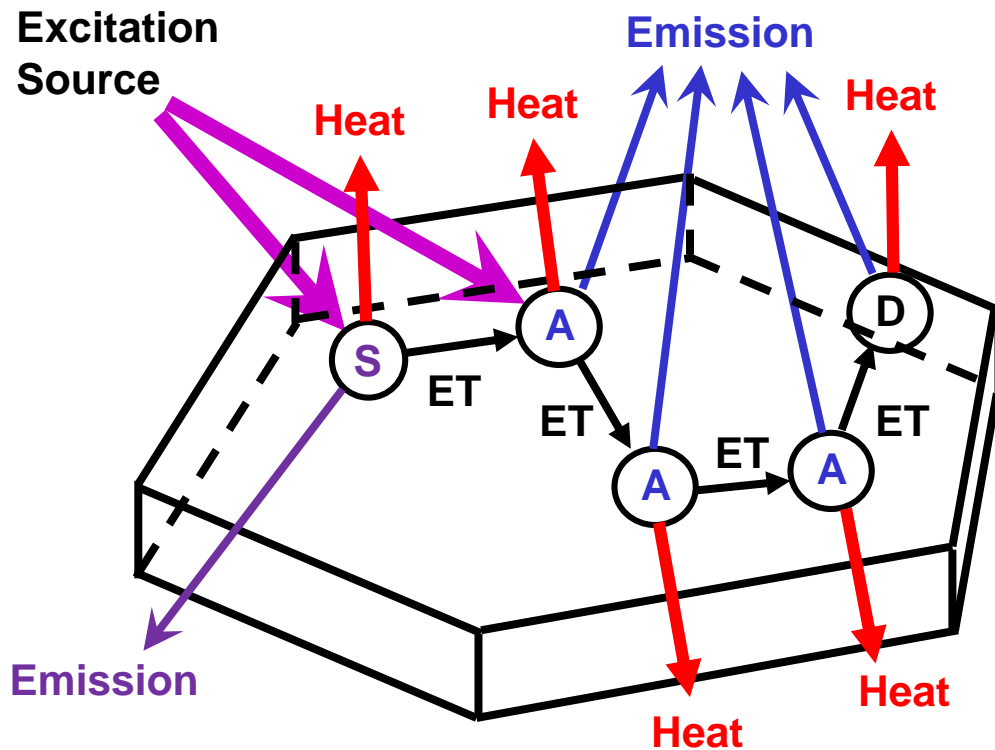
Upon excitation by electrons or UV radiation



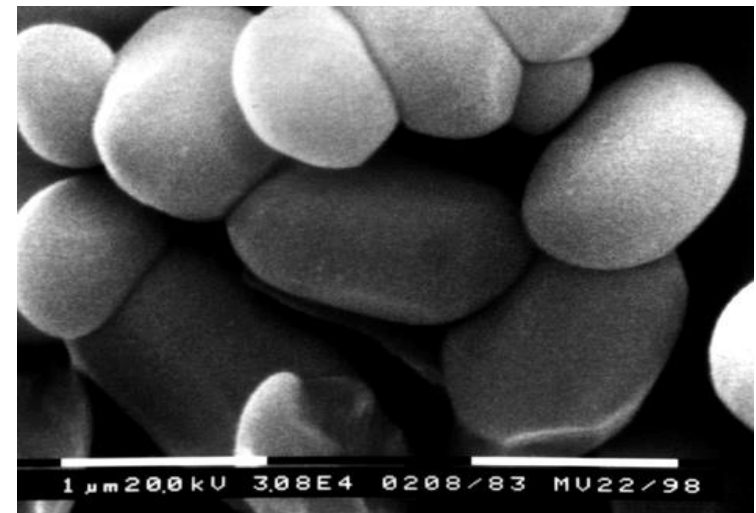
7.2 Definition and Working Principle

Working principle

1. **Excitation:** Absorption of energy from an external source
2. **Energy transfer (ET):** To activator ions (luminescence) or defects (storage)
3. **Relaxation :**
Radiative: Emission (luminescence) → Luminescent pigm.
Non-radiative: Heat (phonons) → Pigment



SEM image of $(Y,Gd)BO_3:Eu$



Typical particle size 1 - 10 μm

7.3 Luminescence Mechanisms

| Type | Physical process (time scale) |
|--------------------------------|---|
| Fluorescence | Spin-allowed transition (ns - μ s) |
| Phosphorescence | Spin-forbidden transition (ms) |
| Afterglow (pers. luminescence) | Thermal activation of charge carriers (s) |

| Type | Excitation source | Applications |
|---------------------|---|------------------------|
| Photoluminescence | UV photons | Fluorescent lamps |
| Radioluminescence | x- and γ -rays, e^- , e^+ , α | x-ray imaging, CT, PET |
| Cathodoluminescence | Electrons | TVs, monitors |
| Electroluminescence | Electric field | LEDs, EL displays |
| Thermoluminescence | Heat | Age determination |
| Chemiluminescence | Chemical reaction | Emergency signals |
| Bioluminescence | Biochemical reaction | Fireflies, jellyfish |
| Sonoluminescence | Ultrasound | - |
| Mechanoluminescence | Mechanical energy | - |
| Lyoluminescence | Free radicals | - |

7.4 Chemical Composition

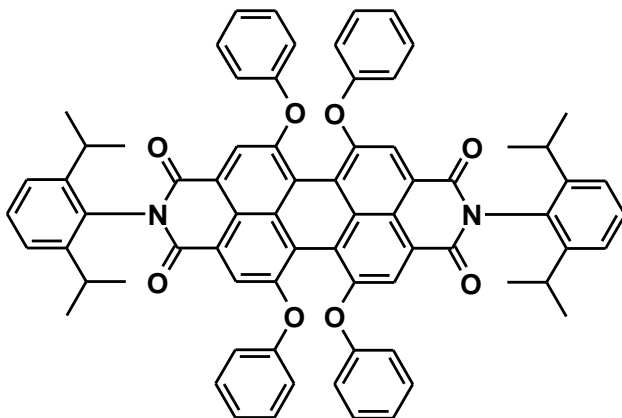
Organic phosphors (dyes or pigments)

Requirements and properties

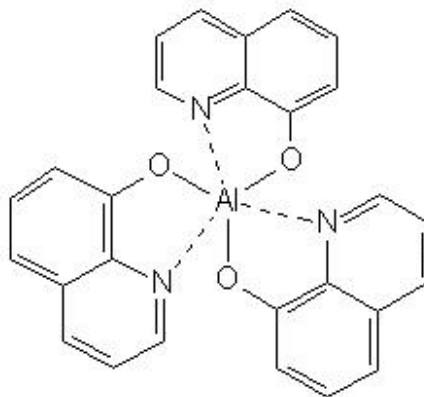
- usually aromatic compounds: No C-H, N-H, or O-H bonds as $\nu > 2900 \text{ cm}^{-1}$ yields MPR
- low energy $\pi \rightarrow \pi^*$ transitions
- quantum yield increases with number of aromatic rings and degree of condensation
- fluorescence especially favored for rigid structures
- fluorescence increase for bounding to a metal \rightarrow complex formation

Examples of selected efficient fluorescent compounds

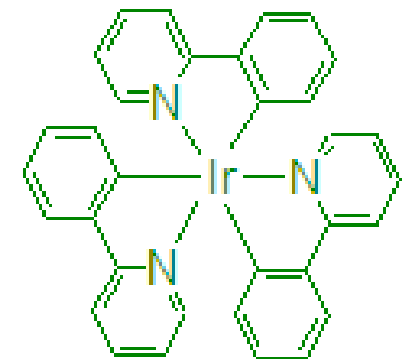
Perylenes



[Al(8-hydroxyquinolate)₃]



[Ir(phenylpyridine)₃]

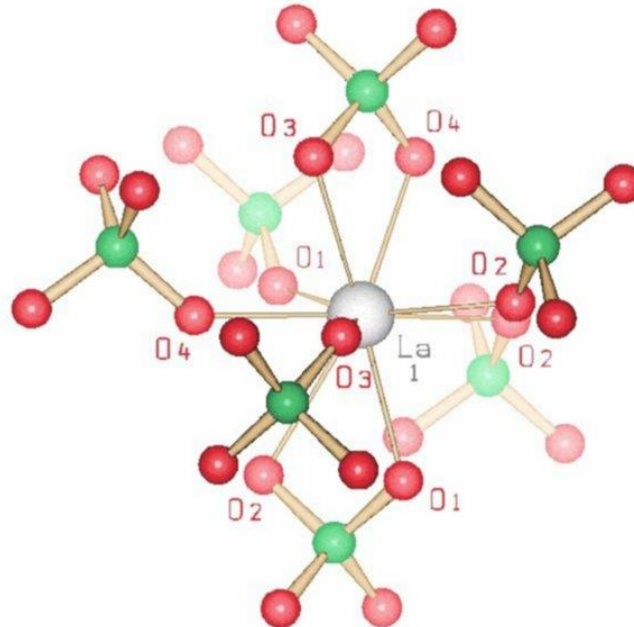


7.4 Chemical Composition

Inorganic phosphors (Luminescent pigments)

- Host material + Dopants + Defects
- Dopants = Activators + Sensitizers + Impurities
- Defects = 0-D (vacancies), 1-D (dislocations), 2-D (boundaries, surfaces), 3-D (pores)

Example, writings: $\text{La}_{1-x-y}\text{Ce}_x\text{Tb}_y(\text{PO}_4) = (\text{La,Ce,Tb})\text{PO}_4 = \text{LaPO}_4:\text{Ce,Tb} = \text{Ce,Tb}:\text{LaPO}_4$



7.4 Chemical Composition

Composition \Rightarrow **Inorganic host** + **Dopants** + **Impurities** + **Defects**

Dopants = Activator/Sensitizers (Impurities) = RE-, TM-, and s²-ions

| | | | | | | | | | | | | | | | | | | |
|----|----|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|-----|-----|-----|-----|----|---|
| 1 | | | | | | | | | | | | | | | | | 18 | |
| 1 | 2 | | | | | | | | | | | 13 | 14 | 15 | 16 | 17 | 2 | 1 |
| H | | | | | | | | | | | | B | C | N | O | F | He | |
| 3 | 4 | | | | | | | | | | | 5 | 6 | 7 | 8 | 9 | 10 | 2 |
| Li | Be | | | | | | | | | | | Al | Si | P | S | Cl | Ne | |
| 11 | 12 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 3 |
| Na | Mg | | | | | | | | | | | Al | Si | P | S | Cl | Ar | |
| 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 4 |
| K | Ca | Sc | Ti | V | Cr | Mn | Fe | Co | Ni | Cu | Zn | Ga | Ge | As | Se | Br | Kr | |
| 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 5 |
| Rb | Sr | Y | Zr | Nb | Mo | Tc | Ru | Rh | Pd | Ag | Cd | In | Sn | Sb | Te | I | Xe | |
| 55 | 56 | 57 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 6 |
| Cs | Ba | La | Hf | Ta | W | Re | Os | Ir | Pt | Au | Hg | Tl | Pb | Bi | Po | At | Rn | |
| 87 | 88 | 89 | 104 | 105 | 106 | 107 | 108 | 109 | 110 | 111 | 112 | | | | | | 7 | |
| Fr | Ra | Ac | Rf | Db | Sg | Bh | Hs | Mt | Ds | Rg | Cn | | | | | | | |
| | | | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 6 | |
| | | | Ce | Pr | Nd | Pm | Sm | Eu | Gd | Tb | Dy | Ho | Er | Tm | Yb | Lu | | |
| | | | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 | 101 | 102 | 103 | 7 | |
| | | | Th | Pa | U | Np | Pu | Am | Cm | Bk | Cf | Es | Fm | Md | No | Lr | | |

7.4 Chemical Composition

Luminescent pigment = Inorganic host + Dopants (Impurities) + Defects

Inorganic Host

- Selection in accordance to requirements defined by the application area:
Excitation energy, absorption strength, chemical environment, temperature, pressure and so on

Dopants (Impurities)

- Selection and concentration depends on host lattice and application:
Solubility, mobility, oxidation state stability, CT state location
- Co-dopants to enhance absorption

Defects

- Afterglow (persistent luminescence)
- Luminescence quenching (conc. and temperature dependent)
- Stability reduction

7.4 Chemical Composition

Luminescent pigment = Inorganic host + Dopants (s^{2-} , TM, or RE ions) + Defects

Inorganic host

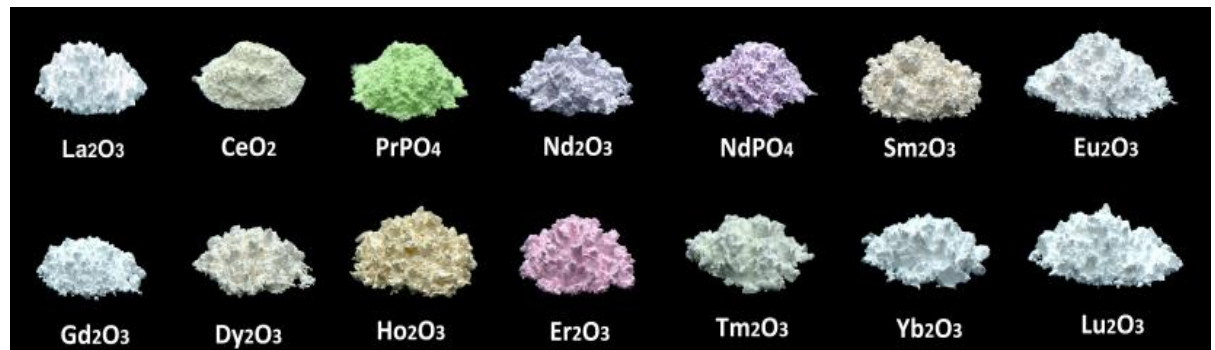
- Oxides Y_2O_3 , $Y_3Al_5O_{12}$, YBO_3 , YVO_4 , YPO_4 , $LaPO_4$, $BaMgAl_{10}O_{17}$, ...
- Sulfides ZnS , MgS , CaS , SrS , $SrGa_2S_4$, $SrIn_2S_4$, Y_2O_2S , Gd_2O_2S , ...
- Fluorides CaF_2 , $LiYF_4$, K_2SiF_6 , KYF_4 , KY_3F_{10} , YOF , K_2NbF_7 , ...
- Nitrides $CaSiN_2$, $CaAlSiN_3$, $Sr_2Si_5N_8$, $La_3Si_6N_{11}$, $SrSi_2N_2O_2$, $SrLiAl_3N_4$, ...

Dopants (impurities)

- s^2 Ions Sn^{2+} , Sb^{3+} , Tl^+ , Pb^{2+} , Bi^{3+}
- TM Ions Ti^{3+} , $V^{2+/3+}$, $Cr^{3+/4+}$, $Mn^{2+/4+}$, Fe^{3+} , Co^{2+} , Ni^{2+} , Cu^{+2+} , Ag^+ , Au^+
- RE Ions Ce^{3+} , Pr^{3+} , Nd^{3+} , $Sm^{2+/3+}$, $Eu^{2+/3+}$, Gd^{3+} , Tb^{3+} , Dy^{3+} , Er^{3+} , Tm^{3+} , $Yb^{2+/3+}$

Defects

- Cation vacancy V_C
- Anion vacancy V_A
- Interstitials I
- Colour centers F



7.4 Chemical Composition

Luminescent pigment = Inorganic host + Dopants (impurities) + Defects

Inorganic Host

- Coordination number and geometry
- Symmetry of activator sites
- Optical band gap
- Phonon spectrum

Dopants (impurities) and defects

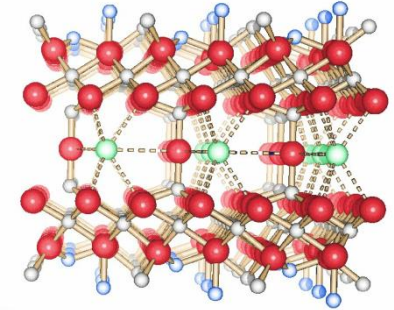
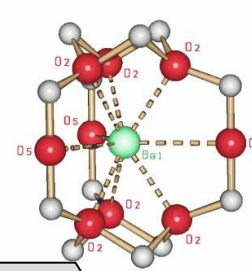
- Concentration
- Phase diagram and miscibility gaps

Particle surface

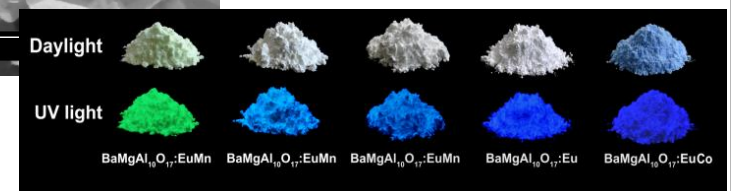
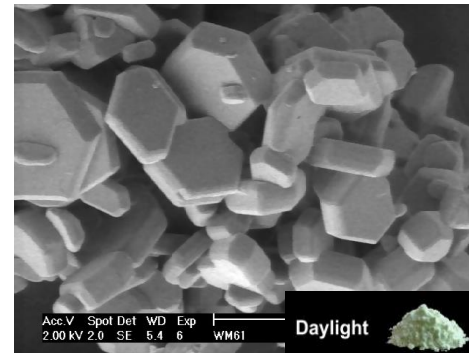
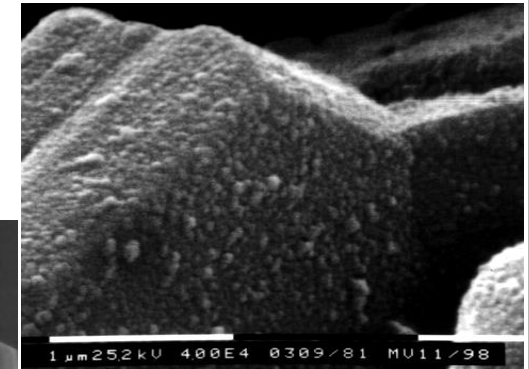
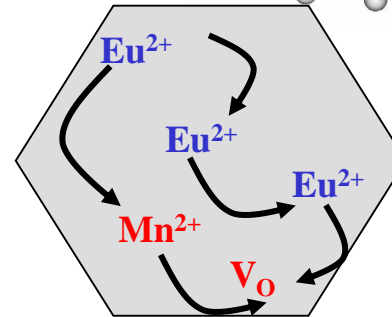
- Zeta-potential
- Surface area, defects, and energy
- Coatings → Light in- and outcoupling

Particle morphology

- Shape
- Particle size distribution
- Agglomeration



SCHAKAL



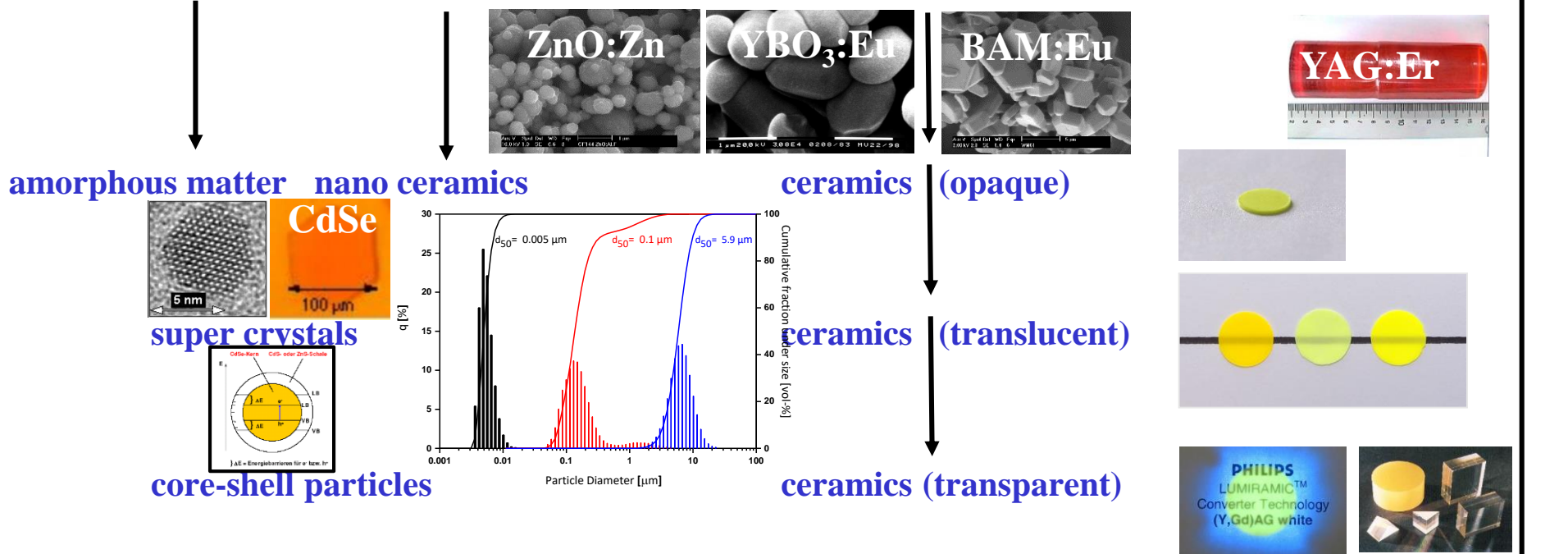
7.4 Chemical Composition

Luminescent pigments – Particle Morphology and Surface Optimisation

Novel application areas ← → lifetime↑, $T_{1/2}$ ↑, α ↓, λ ↑

0.1 nm 1 nm 10 nm 100 nm 1 μ m 10 μ m 100 μ m 1 mm 10 mm 100 mm

atoms cluster nano crystals micro crystals single crystals



Glasses CdSe, CsPbBr₃
(bio)marker, μ -LEDs, PV units

(Y,Tb,Lu)₃Al₅O₁₂:Ce Lu₃Al₅O₁₂:SE (SE = Ce - Yb)
high power LEDs/laser, scintillators, excimer lamps

7.5 Composition and Function

Most relevant physical properties

Photoluminescence (PL) spectra

Absorption and reflection spectra

Quantum yield (QY) (internal and external)

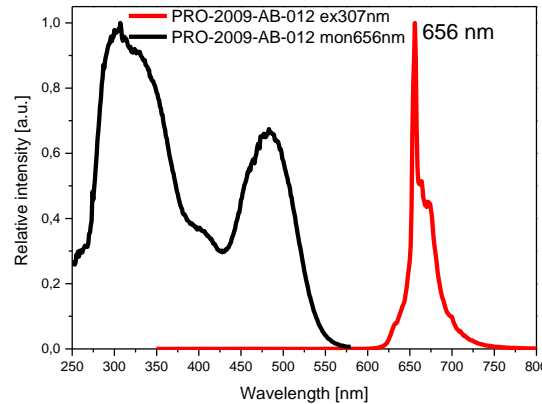
Stability and colour point consistency

Decay curves and afterglow (T-dependent)

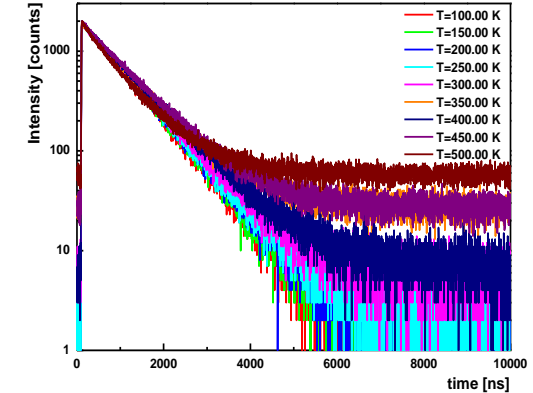
Thermal quenching

Linearity (saturation)

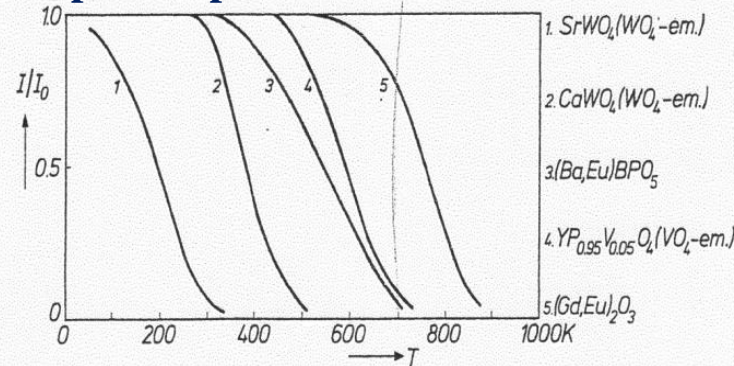
PL (excitation and emission) spectrum of $\text{Mg}_2\text{TiO}_4:\text{Mn}$



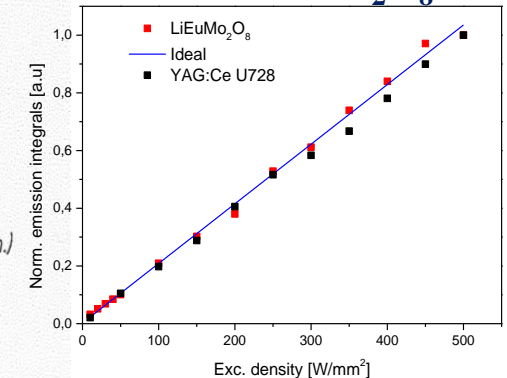
Decay curves of $\text{SrSi}_2\text{N}_2\text{O}_2:\text{Eu}$



Temperature dependent PL of some phosphors upon 254 nm excitation



Linearity of $\text{YAG}:\text{Ce}$ and $\text{LiEuMo}_2\text{O}_8$



7.5 Composition and Function

Composition: Listed by the activator (transition metal ions and s²-ions)

| Activator | Host material | Emission at [nm] | Color | Applications |
|------------------------|--|------------------|-----------------|--|
| Cr³⁺ | Al ₂ O ₃ (Ruby) Ga ₃ Ga ₅ O ₁₂ (Garnet) | 694 | Red IR-A | Solid State Laser NIR LEDs |
| Mn²⁺ | Zn ₂ SiO ₄ (Willemite) BaMgAl ₁₀ O ₁₇ (β-Alumina) | 525 515 | Green Green | PDPs, CRTs PDPs, FLs |
| Mn⁴⁺ | Mg ₄ GeO _{5.5} F K ₂ SiF ₆ | 655 630 | Deep Red Red | Hg high-pressure lamps LEDs |
| Fe³⁺ | LiAlO ₂ | 735 | Red | FLs |
| Cu⁺ | ZnS | 530 | Green | CRTs |
| Ag⁺ | ZnS | 450 | Blue | CRTs |
| Sn²⁺ | (Sr,Mg) ₃ (PO ₄) ₂ | 630 | Red | Hg high-pressure lamps |
| Sb³⁺ | (Sr,Ca) ₅ (PO ₄) ₃ (Cl,F) | 480 | Blue-Green | FLs |
| Tl⁺ | NaI CsI | 415 560 | Blue Yellow | x/γ-ray detectors x/γ-ray detectors |
| Pb²⁺ | BaSi ₂ O ₅ (Sanbornite) Sr ₂ MgSi ₂ O ₇ (Akermanite) | 350 365 | UV-A UV-A | FLs for tanning |
| Bi³⁺ | Bi ₄ Ge ₃ O ₁₂ | 480 | Blue-Green | x/γ-ray detectors |

7.5 Composition and Function

Composition: Listed by the activator (rare earth ions: Ce ... Yb)

| Activator | Host material | Emission at (nm) | Color | Applications |
|------------------|--|------------------|------------|--|
| Ce ³⁺ | LaPO ₄ | 320 | UV-B | FLs for tanning |
| | YPO ₄ | 335, 355 | UV-A | FLs for tanning |
| | Y ₃ Al ₅ O ₁₂ (Garnet) | 560 | Yellow | FLs, LEDs |
| Pr ³⁺ | Gd ₂ O ₂ S | 510 | Green | Computer Tomography (CT) |
| | CaTiO ₃ | 610 | Red | Field Emission Displays (FEDs) |
| Nd ³⁺ | Y ₃ Al ₅ O ₁₂ (Garnet) | 1064 | IR-A | Solid State Laser |
| Eu ²⁺ | SrB ₄ O ₇ | 368 | UV-A | FLs for tanning |
| | BaMgAl ₁₀ O ₁₇ | 453 | Blue | FLs, PDPs |
| | Sr ₄ Al ₁₄ O ₂₅ | 490 | Blue-green | FLs, LEDs |
| Eu ³⁺ | Y ₂ O ₃ | 611 | Red | FLs |
| | YVO ₄ | 615 | Red | Hg high-pressure lamps |
| Gd ³⁺ | (La,Bi)B ₃ O ₆ | 311 | UV-B | FLs for photochemistry and photomedicine |
| | Lu ₃ Al ₅ O ₁₂ (Garnet) | 314 | UV-B | |
| Tb ³⁺ | LaPO ₄ | 544 | Green | FLs |
| | CeMgAl ₁₁ O ₁₉ | 544 | Green | FLs |
| | (Gd,Ce)MgB ₅ O ₁₀ | 544 | Green | FLs |
| Yb ³⁺ | Y ₃ Al ₅ O ₁₂ (Garnet) | 980 | IR-A | Solid State Laser |

7.6 Application Areas

Application in

Excitation source

Scintillator crystals

X-rays intensifier

Cathode ray tubes

Plasma screens

Xe-discharge lamps

Hg-high pressure discharge lamps

Hg-low pressure discharge lamps (FLs)

Emissive LCDs

Phosphor converted light emitting diodes (pcLEDs)

Solid State Laser (SSL)

$\text{Lu}_3\text{Al}_5\text{O}_{12}:\text{Pr}$ (EOT)



γ -rays, particles

X-rays

electrons

147, 172 nm

172 nm

200 – 350 nm

185, 254 nm

370 – 400 nm

370 – 480 nm

300 – 1000 nm

Host material Activator
 ↑
 Excitation energy

Main application areas: Lighting, imaging, projection, detection, and sensing

7.6 Application Areas

TV



LEDs



Plasma TVs



Electroluminescent screens

Fluorescent lamps



Tomography



7.6 Application Areas

| Function | Application field |
|---|---|
| Optical brighteners | Paint, paper, pulp, clothing, detergent |
| Copy protection | Banknotes, stamps, credit cards, certificates, tickets |
| Product protection | Pharmaceuticals, plastics |
| Security labeling | Emergency exit lighting, emergency exits |
| Advertising / visualization | Decoration, advertisement, logos |
| Conversion of high-energy radiation or particles | X-ray films, CT, SPECT, positron emission tomography, EUV-amplifier |
| Cosmetics | Dental ceramics, tanning lamps |
| Marker for the analysis | Detection of nucleic acids + proteins |
| Lithographie | Photocopier |
| Photochemistry and biology | Water purification, disinfection, breeding boxes and cabinets, air pollution control |
| Medicine | Diagnostics, photodynamic therapy |

7.7 Band Emitting Phosphors

Optical transitions (mostly interconfigurational)

- Charge-Transfer (LMCT or MLCT)
- $5s^2-5s^15p^1$, $6s^2-6s^16p^1$
- $4f^n-4f^{n-1}5d^1$
- $3d^n-3d^n$

↑
FWHM

Suitable activator ions/moieties

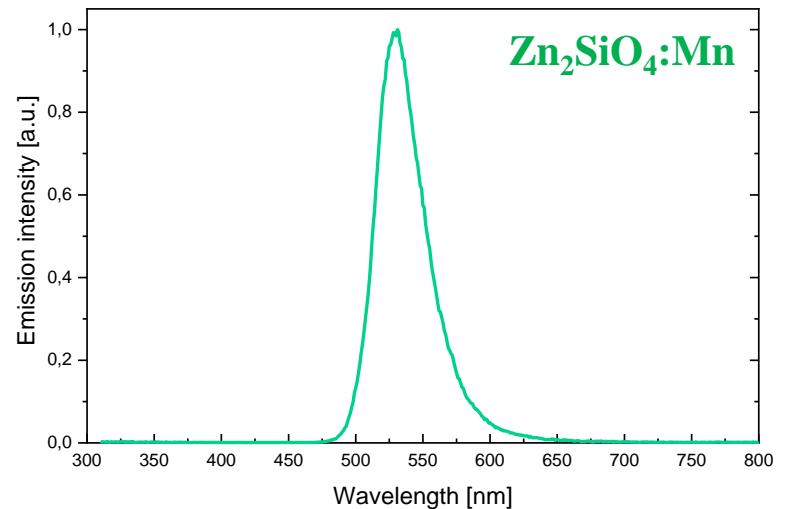
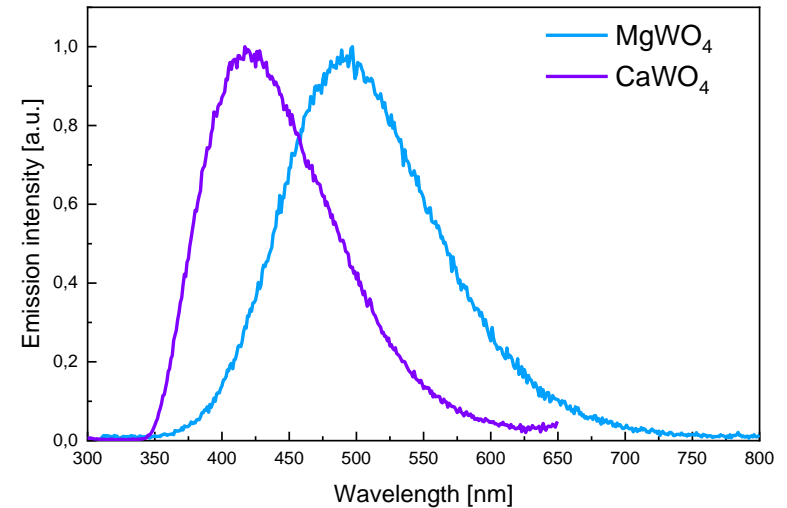
- VO_4^{3-} , WO_4^{2-}
- Sn^{2+} , Sb^{3+} , Tl^+ , Pb^{2+} , Bi^{3+}
- Ce^{3+} , Eu^{2+} , Yb^{2+}
- Mn^{2+} , Cu^{2+}

Examples

- $(\text{Zn,Be})_2\text{SiO}_4:\text{Mn}$
- CaWO_4
- MgWO_4
- $\text{Ca}_5(\text{PO}_4)_3(\text{F,Cl}):\text{Sb,Mn}$

Mineral type

- Willemite
- Scheelite
- Wolframite
- Apatite

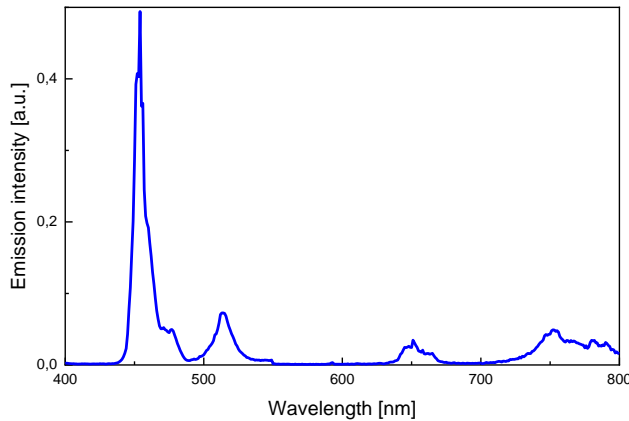
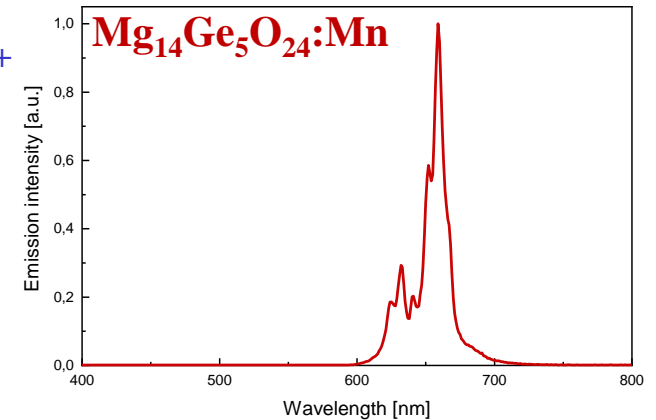


7.8 Line Emitting Phosphors

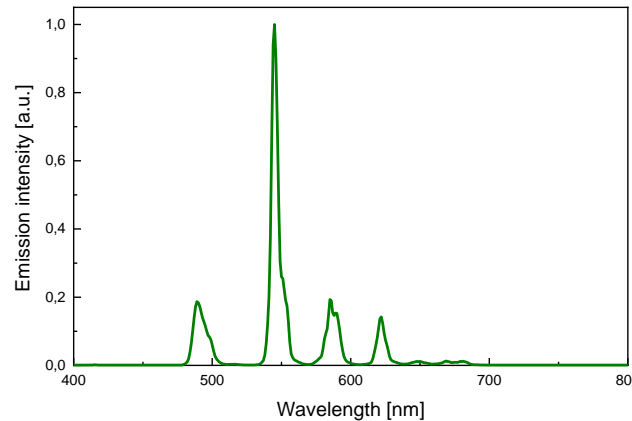
Optical transitions (intraconfigurational)

- $4f^n-4f^n$ **Pr³⁺, Sm³⁺, Eu³⁺, Tb³⁺, Er³⁺, Dy³⁺, Tm³⁺**
- $3d^n-3d^n$ **Mn⁴⁺, Cr³⁺**

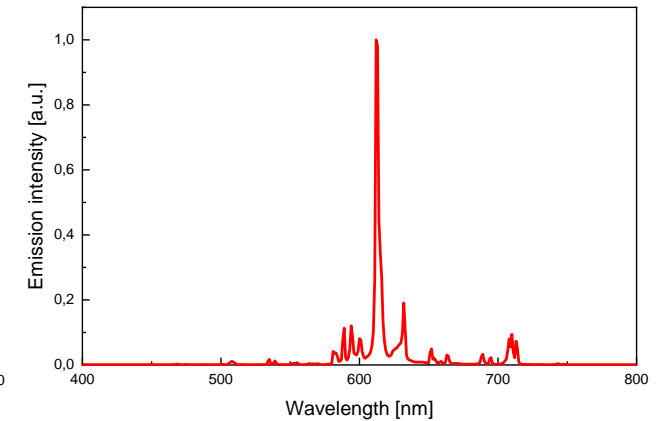
- **Very weak electron-phonon coupling**
- **Lines or line multiplets**



LaPO₄:Tm
YBO₃:Tm
Y₃Al₅O₁₂:Tm



LaPO₄:Ce,Tb
LaMgB₅O₁₀:Ce,Tb
LaMgAl₁₁O₁₉:Ce,Tb



Y₂O₃:Eu
(Y,Gd)BO₃:Eu
YVO₄:Eu

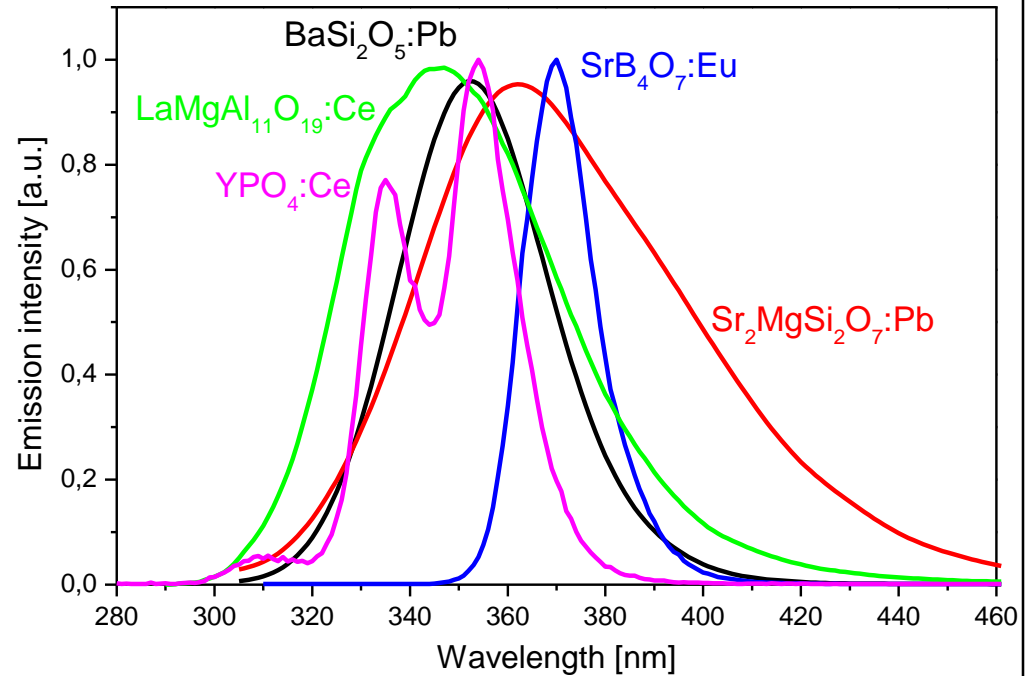
7.9 UV-A Phosphors

Optical transitions

- $4f^n-4f^{n-1}5d^1$
- $6s^2-6s^16p^1$

Suitable activators

- Eu^{2+} >365 nm, 1 band
- Ce^{3+} 2 overlapping bands
- Pb^{2+} 1 very broad band



| Commercial materials | Emission at | Mineral type | Application area |
|--|-------------|-----------------|-------------------|
| • $\text{LaMgAl}_{11}\text{O}_{19}:\text{Ce}$ | 345 nm | Magnetoplumbite | Tanning lamps |
| • $\text{YPO}_4:\text{Ce}$ | 335, 355 nm | Xenotime | Tanning lamps |
| • $\text{BaSi}_2\text{O}_5:\text{Pb}$ | 350 nm | Sanbornit | Tanning lamps |
| • $\text{Sr}_2\text{MgSi}_2\text{O}_7:\text{Pb}$ | 365 nm | Akermanite | Tanning lamps |
| • $\text{SrB}_4\text{O}_7:\text{Eu}$ | 368 nm | Borax | Black light lamps |

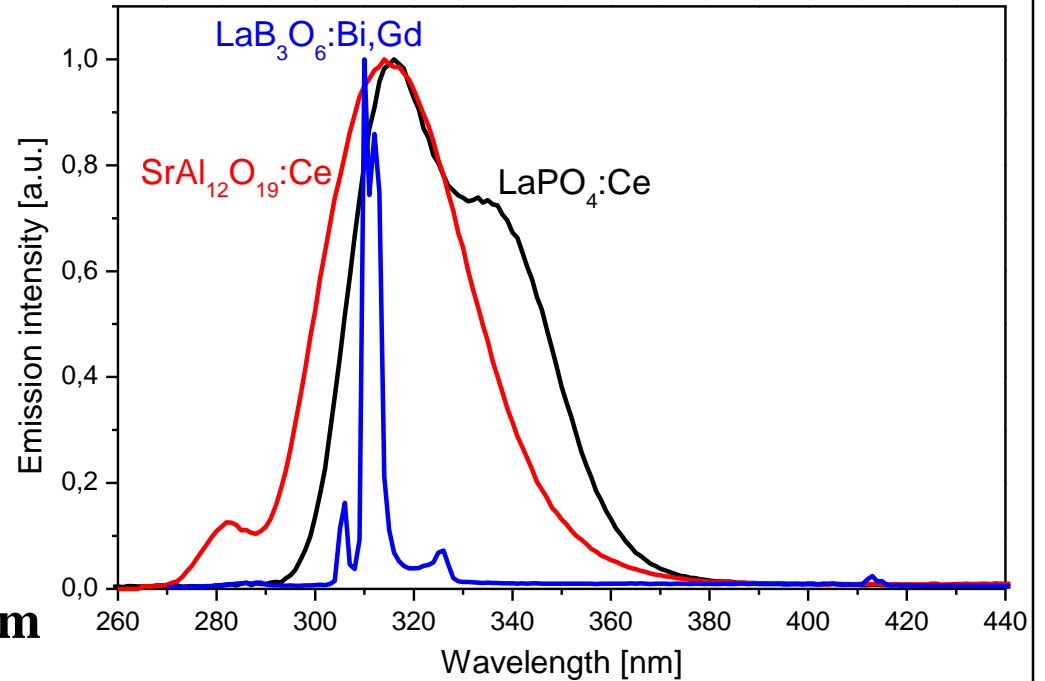
7.10 UV-B Phosphors

Optical transitions

- $4f^n-4f^{n-1}5d^1$
- $4f^n-4f^n$
- $6s^2-6s^16p^1$

Suitable activators

- Ce^{3+} 2 overlapping bands
- Bi^{3+} 1 band
- Gd^{3+} few lines around 312 nm



| <u>Commercial materials</u> | <u>Emission at</u> | <u>Mineral type</u> | <u>Application area</u> |
|-----------------------------|--------------------|---------------------|-------------------------|
| • $SrAl_{12}O_{19}:Ce$ | 300 nm | Magnetoplumbite | Tanning lamps |
| • $LaB_3O_6:Bi,Gd$ | 311 nm | - | Medical lamps |
| • $LaPO_4:Ce$ | 320 nm | Monazite | Tanning lamps |

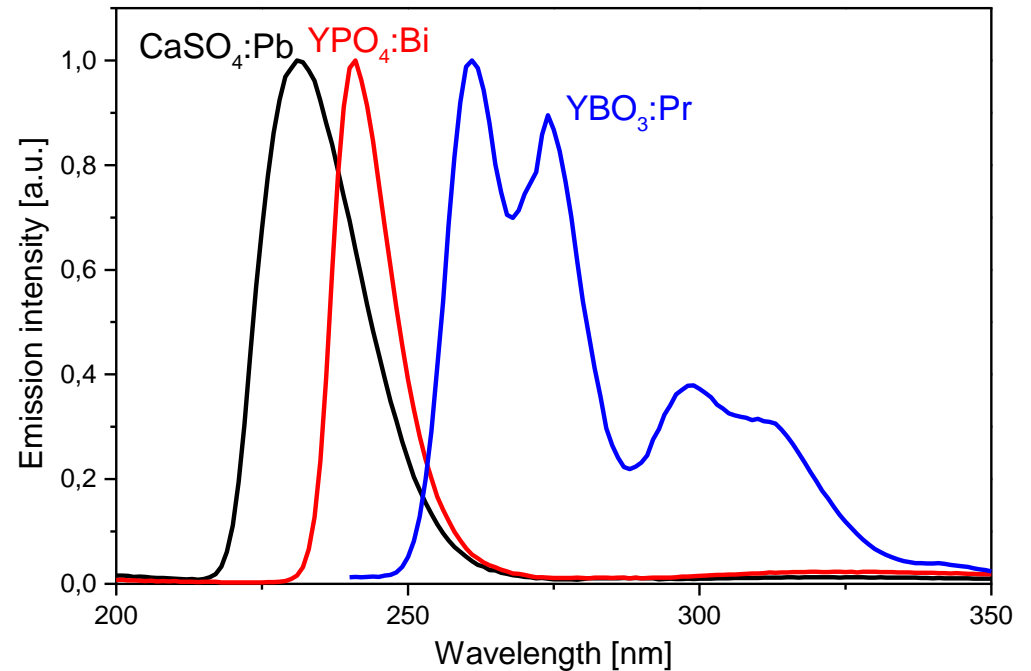
7.11 UV-C Phosphors

Optical transitions

- $4f^n-4f^{n-1}5d^1$
- $6s^2-6s^16p^1$

Suitable activators

- Pr^{3+} **4 overlapping bands**
- Tl^+ **1 band**
- Pb^{2+} **1 band**
- Bi^{3+} **1 band**



| Examples | Emission at | Mineral type | Application area |
|-----------------------------|--------------------|---------------------|-------------------------|
| • $\text{YBO}_3:\text{Pr}$ | 265 nm | Vaterite | Disinfection |
| • $\text{YAlO}_3:\text{Pr}$ | 245 nm | Perovskite | Disinfection |
| • $\text{YPO}_4:\text{Bi}$ | 240 nm | Xenotime | Disinfection |
| • $\text{CaSO}_4:\text{Pb}$ | 230 nm | Anhydrite | Disinfection |

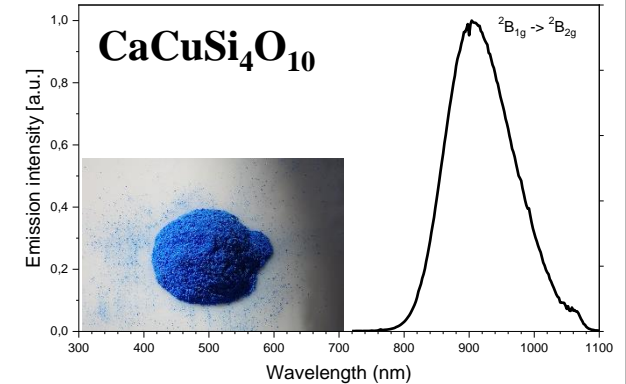
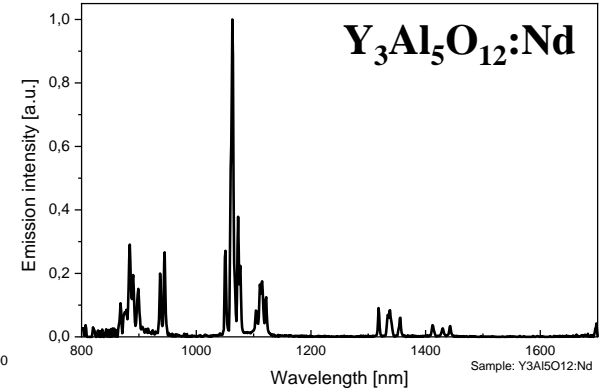
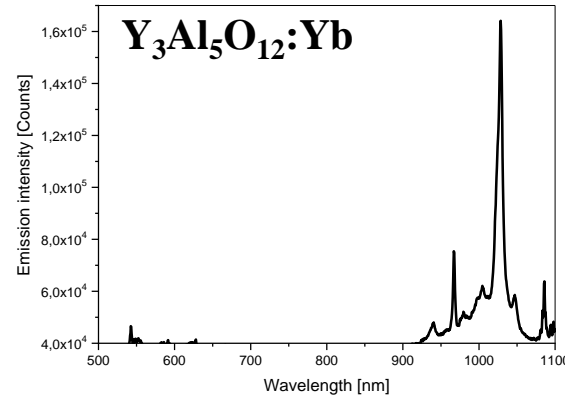
7.12 NIR Phosphors

Optical transitions

- $3d^n-3d^n$
- $4f^n-4f^n$

Suitable activators

- Cr^{3+} lines, 680 - 800 nm
- Cu^{2+} 1 band, 800 - 1000 nm
- Nd^{3+} lines, 1050 - 1100 nm
- Yb^{3+} lines, 950 - 1050 nm



Examples

| Examples | Emission at | Mineral type | Application area |
|--|-------------|--------------|-------------------|
| • Al ₂ O ₃ :Cr | 694 nm | Corundum | Ruby laser |
| • CaCuSi ₄ O ₁₀ | 910 nm | Cuprorivaite | NIR Marker |
| • Y ₃ Al ₅ O ₁₂ :Yb | 1028 nm | Garnet | Solid state laser |
| • Y ₃ Al ₅ O ₁₂ :Nd | 1064 nm | Garnet | Solid state laser |

7.13 Future Trends

Efficiency: Light sources & displays

- External quantum yield (EQY) ↑
- μ -particles → ceramics → single crystals

Lifetime/stability: Light sources & displays

- Defect density ↓ and particle coatings

Miniaturisation: μ -LED (displays)

- PSD ↓: Nanocrystals & Quantum Dots
- Stability ↑: Core-shell particles

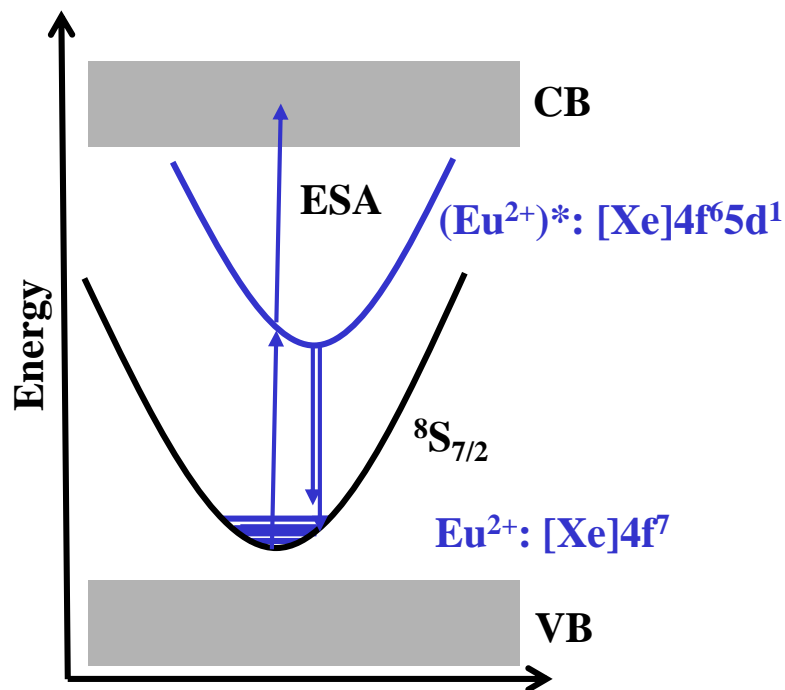
Power density: HP LEDs & laser diodes

- Decay time & ESA ↓ redox stability ↑
- Density of optical center $N_{\text{activator}} [\text{cm}^{-3}]$ ↑

Novel spectra: NUV, NIR, human centric lighting

- UV: (Al,Ga)N LED / Xe excimer lamps
- NIR: (In,Ga)N LED + deep red/NIR emitter

$$\text{EQY} = \frac{\text{Number of emitted photons}}{\text{Number of absorbed photons}}$$



$$P_{\text{em,max}} = N_{\text{activator}} C_{\text{extraction}} R / \tau_r \quad (R = \text{radius})$$

Ref.: Brils Modell, A. Brill, Physica 15 (1949) 361